OTEP-2

Optimal Train Energy Programme mark 2

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This is essentially a user's guide to the application programme developed by Mr. S. B. Huxford under the authors' supervision. A secondary purpose of this report is to document the programme for further developments. A detailed discussion of the problem of optimizing train energy and its solution is given elsewhere ([1]).

1. Introduction

OTEP is an application programme calculating and providing information about several candidates for the solution of a problem in optimal control theory seeking to minimize the energy used by a vehicle performing a given journey in a prescribed time.

The work on the project started during the second author's (N.K.N.) visit to Flinders University in August-November, 1981 when the authors met a group of researchers at the South Australian Institute of Technology solving similar problems by different means ([2],[3],[5]). The methodology of the present programme - solving a non-linear boundary value problem for a system of ordinary differential equations by PASVAR3 ([4]) - was tested by a simpler application and the basic theory and algorithmic approach developed.

The OTEP itself was programmed in 1982 by Mr. S. Huxford and, to some extent, also by Dr. M. Lohe. Both N.K.N.'s visit and the programming assistance were supported by Flinders University research grants. All the contributions are gratefully acknowledged.

The OTEP-2 is an extension of OTEP-1 in two areas:

(i) a more general - speed dependent - control component of the differential equation describing the motion of the vehicle and

(ii) a wider range of special extremal problems solved.

The first aspect was motivated by the analysis of experimental results of accelerating and braking performance tests [6]. The package can now produce very realistic journey profiles. The second extension, allowing in particular to calculate journeys with prescribed - rather than optimal - holding speed, gives greater flexibility in estimating the costs of non-optimal strategies.

2. The problem

Given constants $\sigma, T > 0$, $u_A > 0 > u_B$ and (sufficiently smooth) functions g defined on $[0,\sigma]$ and p , p_A , p_B defined on $[0,\infty)$ the problem is to find a piece-wise continuous function u on $[0,T]$ minimizing

$$J(u) := \int_0^T u_+(t)\dot{x}(t)p_A(\dot{x}(t))dt \qquad (1)$$

where x satisfies

$$\ddot{x}(t) = u(t)p_u(\dot{x}(t)) - p(\dot{x}(t)) - g(x(t)) \quad , \quad t \in (0,T) \ , \qquad (2)$$

$$x(0) = \dot{x}(0) = \dot{x}(T) = 0 \quad , \quad x(T) = \sigma \ , \qquad (3)$$

$$\dot{x}(t) \geq 0 \quad , \quad t \in [0,T] \ , \qquad (4)$$

Here $p_u = p_A$ for $u > 0$ and $p_u = p_B$ for $u < 0$, and u satisfies

$$u_B \leq u(t) \leq u_A \quad , \quad t \in [0,T] \ . \qquad (5)$$

Also $u_+(t) = \frac{1}{2}(u(t) + |u(t)|)$, $\dot{x}(t)$ denotes the derivative with respect to t, which is assumed continuous, while the second derivative $\ddot{x}$ may be discontinuous where u is. In this problem,

$x(t)$	is the distance of the vehicle at time t
$\dot{x}(t)$	its speed
$\ddot{x}(t)$	its acceleration
σ	the total distance of the journey
T	the total time of the journey
$p(\dot{x})$	the deceleration due to speed (positive non-decreasing)
$p_A(\dot{x})$	speed dependent component of the tractive effort during acceleration (positive non-increasing)
$p_B(\dot{x})$	speed dependent component of the tractive effort during braking (positive for the required range of velocities)
$g(x)$	the deceleration due to the properties of the track
u	the demanded acceleration (the control variable)
J	the energy equivalent criterion of performance.

The definition (1) of J measures the energy input causing the acceleration of the vehicle but assumes that the braking - when $u < 0$ - is free, i.e. it neither requires nor produces energy. The differential equation (2) expresses the balance between the factors contributing to the acceleration of the vehicle and is to be satisfied for all but a finite number of times where the control u may be discontinuous. The boundary conditions (3) express the fact that the vehicle is stationary at the beginning and end of the journey of length σ completed in time T. The constraint (4) forbids the vehicle to move backwards (to be expected for minimizing the energy), while the constraint (5) expresses the limitations of the brakes and the engine of the vehicle.

In [1] we show that for a pair of functions u, x satisfying (2)-(5) to minimize the functional J, the control function u must be piecewise equal to either u_A or u_B or zero or to be such that the speed $\dot{x}$ is constant. We call the intervals where these strategies occur accelerating (A), braking (B), coasting (C) and maintaining (M) periods, respectively. In more detail if λ_1, λ_2 are continuous solutions of the co-state system

$$\begin{aligned} \dot{\lambda}_1(t) &= u_+(t)(p_A(\dot{x}) + \dot{x}p_A'(\dot{x})) + (p'(\dot{x}) - up_u'(\dot{x}))\lambda_1(t) - \lambda_2(t) \\ \dot{\lambda}_2(t) &= \frac{dg(x(t))}{dx}\lambda_1(t) \end{aligned} \qquad (6)$$

the necessary conditions for optimality are:

$$\begin{array}{lll} \lambda_1(t) \geq \dot{x}(t) & \text{on A period} & (u(t) \equiv u_A) \\ \lambda_1(t) = \dot{x}(t) & \text{on M period} & (0 \leq u(t) \leq u_A) \\ 0 \leq \lambda_1(t) \leq \dot{x}(t) & \text{on C period} & (u(t) \equiv 0) \\ \lambda_1(t) \leq 0 & \text{on B period} & (u(t) \equiv u_B) \end{array}$$

It is also shown in [1], [2] that for *flat* ground functions ($g = \text{const.}$) an optimal solution comprised of the above strategies AMCB, in that order, exists for σ not too large with respect to the other parameters of the problem. For σ close to the maximal reachable distance the maintaining period will vanish - we then talk about the ACB strategy.

For arbitrary ground it may be necessary to combine a larger number of the above strategies to satisfy the conditions of optimality. However, by continuity, for a close to flat ground function g , the ACB or AMCB should suffice. OTEP-2 is designed to calculate the ACB and AMCB solutions to the problem and to test the optimality conditions of these solutions.

OTEP accepts the ground function g of a special form: a smoothed piecewise constant function

$$g(x) = (g_1 + g_L)/2 + \frac{1}{\pi}\sum_{j=1}^{L-1}(g_{j+1} - g_j)\arctan\left(\frac{x - z_j}{\varepsilon_G}\right) \tag{7}$$

where $\varepsilon_G > 0$ is the smoothing parameter,

$0 < z_1 < z_2 < \ldots < z_L \geq \sigma$ are the breaking points and

g_j , $j = 1,2,\ldots,L$ are the limiting (for $\varepsilon_G \to 0$) values of g between the breaking points $(x \in (z_{j-1}, z_j) \Rightarrow g(x) \to g_j)$.

Similarly, OTEP-2 accepts the speed dependent functions p_A , p_B and p in a special form - as polynomials of second degree. In particular,

$$\begin{aligned} p_A(v) &= 1 + b_A v + c_A v^2 \\ p_B(v) &= 1 + b_B v + c_B v^2 \\ p(v) &= a + bv + cv^2 \end{aligned} \tag{8}$$

The normalization of the constant terms in p_A and p_B avoids ambiguity as these functions are multiplied by the constants u_A and u_B , respectively. The constants a , b , c are the coefficients of the Davis formula for the rolling and velocity induced resistance of moving vehicles.

3. Range and domains of extremal and optimal problems.

If the tractive effort data u_A , u_B , $p_A(\dot{x})$, $p_B(\dot{x})$ and the resistance functions $p(\dot{x})$ and $g(x)$ are fixed, a number of particular problems may be considered.

In the following discussion we identify such problems and describe the ranges of total times T , total distances σ and speed restrictions v_m for which they have solutions. The abbreviations used indicate the sequence of strategies in question and the goals of the problems (see Table 1).

Table 1 - Abbreviations.

STRATEGIES:

A	accelerating phase	$(u = u_A)$
B	braking phase	$(u = u_B)$
C	coasting phase	$(u = 0)$
V	maintaining a prescribed velocity phase	
M	maintaining an optimal velocity phase	

GOALS:

D	problem determines the distance (the given σ is ignored)
T	problem determines the time (the given T is ignored)
S	if both T and σ are given then the problem determines the speed (the given v_m is ignored) or the speed restriction v_m is given and both T and σ are determined.

(i) Problems with the time T given:

(i.1) The AB/D problem determines the maximal distance $\sigma_{ABD}(T)$ reachable in time T which is achieved by the AB strategy; we denote the maximal speed during such journey $v_{ABD}(T)$.

(i.2) Similarly, the AC/D problem determines the maximal distance $\sigma_{ACD}(T)$ reachable in time T without braking and the corresponding maximal speed $v_{ACD}(T)$.

(i.3) The problem AVB/D determines the maximal distance $\sigma_{AVB}(t,v_m)$ reachable in time T with the speed restriction v_m. For the speed restriction to be active we must have $v_m < v_{ABD}(T)$. We have then $\sigma_{AVB}(T,v_m) < \sigma_{ABD}(T)$.

(i.4) Similarly, the problem AVC/D determines $\sigma_{AVC}(T,v_m)$ assuming $v_m < v_{ACD}(T)$. We have then $\sigma_{AVC}(T,v_m) < \sigma_{AC}(T)$.

(ii) Problems with the distance σ given:

(ii.1) The AB/T problem determines the minimal time $T_{ABT}(\sigma)$ in which it is possible to cover the distance σ; this is achieved by the AB strategy and we denote the maximal speed reached by $v_{ABT}(\sigma)$.

(ii.2) Similarly, the AC/T problem determines the minimal time $T_{ACT}(\sigma)$ needed to cover distance σ without braking and the corresponding maximal speed $v_{ACT}(\sigma)$.

(ii.3) The problem AVB/T determines the minimal time $T_{AVB}(\sigma,v_m)$ needed to cover the distance σ without exceeding the speed v_m; for this restriction to be active we must assume $v_m < v_{ABT}(\sigma)$. We have then $T_{AVB}(\sigma,v_m) > T_{ABT}(\sigma)$.

(ii.4) Similarly, the problem AVC/T determines $T_{AVC}(\sigma,v_m)$ assuming $v_m < v_{ACT}(\sigma)$. We have then $T_{AVC}(\sigma,v_m) > T_{ACT}(\sigma)$.

(iii) Problems with the speed restriction v_m given:

(iii.1) The problem AB/S determines the maximal distance $\sigma_{ABS}(v_m)$ reachable by the AB strategy without exceeding the speed restriction v_m. This will be achieved in time $T_{ABS}(v_m)$.

(iii.2) Similarly, the problem AC/S determines the distance $\sigma_{ACS}(v_m)$ and time $T_{ACS}(v_m)$ of such a journey without braking.

Note that these two problems are not implemented in OTEP-2.

(iv) Problems with both the time T and distance σ given. In all problems in this subsection we must assume

$$T > T_{ABT}(\sigma) \quad \text{or, equivalently,} \quad \sigma < \sigma_{ABD}(T) \qquad (10)$$

so that the distance is reachable in the given time.

(iv.1) The problem AVB/S determines the minimal speed restriction $v_{AVB}(T,\sigma)$ which can be prescribed so that the problem AVCB has a solution. Note that the three relations $T = T_{AVB}(\sigma,v_m)$, $\sigma = \sigma_{AVB}(T,v_m)$ and $v_m = v_{AVB}(T,\sigma)$ are equivalent.

(iv.2) Similarly, the problem AVC/S determines the maximal speed restriction $v_{AVC}(T,\sigma)$ so that the problem AVCB has a solution. As the distance must now be reachable without braking, we must impose a stronger assumption on T and σ :

$$T > T_{ACT}(\sigma) \quad \text{or, equivalently,} \quad \sigma < \sigma_{ACD}(T) \ . \qquad (11)$$

Again, there is an equivalence between relations $T = T_{AVC}(\sigma,v_m)$, $\sigma = \sigma_{AVC}(T,v_m)$ and $v_m = v_{AVC}(T,\sigma)$.

(iv.3) The problem ACB has a unique solution assuming (10) and opposite of (11), i.e.

$$T_{ACT}(\sigma) > T > T_{ABT}(\sigma)$$

or, equivalently, (12)

$$\sigma_{ACD}(T) < \sigma < \sigma_{ABD}(T) \ .$$

We denote its maximal speed $v_{ACB}(T,\sigma)$. Solving for the optimality conditions associated with the ACB problems determines the boolean function $opt_{ACB}(T,\sigma)$ which is true if the ACB solution is optimal.

(iv.4) The problem AMCB has a unique solution assuming (10) and either (11) or, if (11) is false, that $opt_{ACB}(T,\sigma)$ is also false. In other words, the solution will not exist if $opt_{ACB}(T,\sigma)$ is defined and true.

(iv.5) The problem AVCB is the ACB problem with the speed restriction v_m also prescribed. For this constraint to be active we must assume, as mentioned above,

$$v_{AVB}(T,\sigma) < v_m < v_{AVC}(T,\sigma) \ .$$

However, $v_{AVC}(T,\sigma)$ is defined only if (11) holds. Otherwise, the restriction is the speed corresponding to the AVCB strategy without the V section, i.e. ACB which then exists. Summarising, for AVCB problem to have a solution we must assume (10), $v_m > v_{AVB}(T,\sigma)$ and either $v_m < v_{AVC}(T,\sigma)$ or $v_m < v_{ACB}(T,\sigma)$, whichever of the two is defined (decidable by checking (11)).

4. Using OTEP-2.

The programme solves at user's choice the problems defined in the previous section and also, in a standard "RUN", performs the following steps automatically.

(1) (i) Finding the ACB soluiton.

(ii) Checking the optimality of the ACB solution.

(2) (i) Finding the AMCB soluiton.

(ii) Checking the optimality of the AMCB solution.

The programme is essentially menu driven and provides up to 5 levels of output. The input menus and the output options are described in detail in sections 5 and 6, respectively. The basic control flow provides for step 2 to be performed automatically when the ACB solution does not satisfy the optimality condition of step 1(ii).

The user should be aware of the possible shortcomings in the solutions obtained by the PASVA3 package. PASVA3 solves the required systems of ordinary differential equations by a variable order finite difference method with adaptable mesh selection. The user specifies the tolerance (TOL, see section 5) for maximal acceptable errors; however, this may not be achievable due to either strong nonlinearity of the problem, fast changing ground function or simply memory space limitations restricting the allowable number of grid points.

The solutions can be both plotted and tabled. The grid points for the tables are chosen by the package - they are not necessarily equidistant and they are common to all A , C and B periods (see section 6 for details).

We now give two safe sequences of actions for a cautious user wanting to avoid solving problems without meaningful solutions.

The following steps should be followed to find the optimal solution:

step 1 : Solve the AB/T problem and note the minimal time $T_{ABT}(\sigma)$ and speed $v_{ABT}(\sigma)$. If the given $T \le T_{ABT}(\sigma)$ no other problems can be solved - data have to be adjusted and step 1 repeated.

step 2 : Solve the AC/T problem and note the time $T_{ACT}(\sigma)$.

step 3 : If $T \ge T_{ACT}(\sigma)$ proceed to step 6.

step 4 : Solve the ACB problem and note the speed $v_{ACB}(T,\sigma)$ at the switching time between A and C sections.

step 5 : Only if this strategy is not optimal proceed to the next step.

step 6 : Solve the AMCB problem.

The following steps should be followed (or incorporated in the above procedure) to find a solution with prescribed speed restriction v_m :

step 1 : The AB/T problem as above.

step 7 : Solve the AVB/S problem and note the holding speed $v_{AVB}(T,\sigma)$.

step 2 : The AC/T problem as above.

step 8 : If $T \ge T_{ACT}(\sigma)$ proceed to step 10.

step 4 : The ACB problem as above.

step 9 : Choose v_m satisfying $v_{AVB}(T,\sigma) < v_m < v_{ACB}(T,\sigma)$ and proceed to step 12.

step 10: Solve the AVC/S problem and note the holding speed $v_{AVC}(T,\sigma)$.

step 11: Choose v_m satisfying $v_{AVB}(T,\sigma) < v_m < v_{AVC}(T,\sigma)$.

step 12: Solve the AVCB problem.

An experienced user may risk some short cuts. For example, if T is close to $T_{ABT}(\sigma)$ in step 1, then step 2 may be avoided and step 4 proceeded with. On the other hand, if T is much larger than $T_{ABT}(\sigma)$, then step 6 may be directly attempted. Similarly, the decision in step 8 may be guessed or, more directly, a value of v_m chosen and AVCB attempted. The penalty for wrong decisions is that the ODE solver PASVAR will produce, sometimes after using a considerable amount of CPU time, either a meaningless soluiton or a solution not satisfying the constraint $\dot{x} \geq 0$.

We note that in OTEP-2 the speed restriction v_m is applied to the speed reached at the end of the accelerating period (and the following speed holding period) only. The speed reached during the coasting period is not checked; it may exceed v_m on steeper downhill sections. Furthermore, the speed restriction is imposed only in some problems, as specified above.

5. OTEP input.

There are 5 major menus to control the operation of OTEP and to input data to OTEP and several other decision points. Normally, the input is from an interactive terminal (the variables LTI, LTO, determine the logical unit numbers for this purpose; for PRIME these values are both 1). It is also possible to prepare data on a text file of name OTINdd where dd are suitable values for additional logical unit numbers ($07 \le dd \le 20$ or $30 \le dd \le 99$ on PRIME) as demonstrated in Appendix B.

The menus are listed in Appendix A and are mostly self-explanatory. We will now list some specific comments and explanations of various features.

(i) *Main menu* is the central controlling point. Three other menus can be invoked to update parameters describing the problem, controlling the ODE solver PASVAR and those controlling the input and output. The external plot can be initiated (after a successful solution) and the user can stop OTEP in this and some other menus. Fourteen particular problems described in sections 3 and 4 can be initiated and solved; note, however, that the external plot should be attempted only after solving ACB, AVCB or AMCB problems. The standard "RUN" option performs the steps 1(i), 1(ii), 2(i), 2(ii) described in section 4 (last two only if the ACB strategy was not optimal). There are decision points after the steps 1(i) and 2(i) to allow the user to return to the MAIN menu (advisable if the last step was not solved successfully).

(ii) *Case parameters menu* allows the input of new values for the physical constants of the case mentioned in the description of the problem (section 2). The parameters of the ground function g are entered through a sub-menu. All parameters are considered dimensionless. As the output is mostly in fixed point display it is advisable to choose suitable scaling for all units involved.

(iii) *PASVAR parameters menu* allows the specification of the initial and maximal number of grid points to be used by PASVAR and the error tolerance required from the solution. The user is informed if these numbers are chosen incorrectly. The initial number of grid points must be at least 3 but it seems better to set it to 11, e.g.. The maximal number of grid points is limited by the memory available (currently 33) and that for ACB should not exceed the one for AMCB. We note that the optimality problems are always solved on a fixed number points - that used by the preceeding solution.

(iv) *I/O control parameters menu* allows the update of the following switches and logical unit numbers:

(a) Print control switch restricts the output produced by OTEP - value 1 gives the terminal output only, value 2 produces also a user oriented information and value 3 gives also a programmer oriented debugging type output on a separate file. Note that resetting the output levels 2 or 3 erases that level output so far produced.

(b) Interactive I/O logical unit numbers may be reset to allow data input from pre-prepared files and to restrict terminal output during such operation (for details see Appendix B). Any logical unit number n up to 5 will be treated as a terminal identification (i.e. will not be opened or closed) while the others

$(5 < n < 100)$ will be associated with input files OTINn or output files OTOUn.

(c) Level 2 and 3 logical unit numbers may be reset and thus the output directed to other (OTOUxx) files. Again, specifying the same unit number will erase the output so far - on the other hand, changing the logical unit number *before* changing the print control level switch may be used to preserve the output so far accumulated. Note, however, that when exiting from OTEP only the currently specified files are closed - the others, if any, may have to be closed manually by a monitor command.

(d) Two levels of output information are available from PASVAR - brief error messages and detailed progress report. Logical unit number $n = 0$ indicates no output is required; otherwise $(5 < n < 100)$ will direct the output to file OTOUn.

(e) Plot switch p controls the internal plot. Positive value of p causes graphic display of optimal solutions only; negative value will force the plot of the ACB solution regardless of its optimality. $|p|$ determines the graphic display: 0 for no plot, 1 for the plotter-printer (produces files to be spooled later), 2 for Tektronix type graphic terminals, 4 for the DEC-GIGI terminal. The details of the plot display are in the next section.

All input parameters are given default values. These may be inspected by displaying the corresponding menus.

Note that before displaying the MAIN menu and at some other points in the programme an audible signal (beep) is activated. This is to inform the user that the screen is to be erased and new information displayed. However, this will not happen until the user enters the "carriage return" at the terminal (any text will be ignored, may be used for comments); this allows the user to inspect the current display at will.

5. OTEP output.

(i) The level 1 (terminal) output comprises a header identifying the problem solved (e.g., ACB STRATEGY or AMCB STRATEGY-OPTIMALITY, etc.), information on the PASVAR performance and a summary of results. The information on PASVAR performance contains the number of grid points used, the CPU time used, an error estimate and an error flag coded as follows:

0 PASVAR performed satisfactorily.

1 conflicting information on the size of the problem (should not occur in this application).

2 maximal number of grid points not sufficient to reach the required tolerance.

3 Newton method in solving the algebraic non-linear system diverged.

4 the round-off noise level reached in trying to achieve the required error tolerance.

Generally, for non-zero error message codes, the solution may still be meaningful if the error estimate is a small positive number. However, zero error estimate indicates a complete failure of the solution.

The summary of results contains the switching times, distance and speed at these times and the energy required by that section of the journey. For the AMCB strategy the energy for the M (speed maintaining) section is divided into the contributions needed to overcome the speed $(a + b\dot{x} + c\dot{x}^2)$ and ground $(g(x))$ resistances. The maximal and minimal values of u during this section are also displayed (together with distances where they occurred) to provide a quick check for the optimality condition $0 \le u(t) \le u_\Lambda$.

The summary of results of the optimality problems provides two kinds of information: value of $x - \lambda_1$ at the beginning of the C (coasting) section - if this value is positive for the ACB problem then the solution is considered optimal; AMCB problem is attempted only if this value is negative (ACB is definitely not optimal in that case). For the AMCB problem this value is zero by definition and the value displayed provides only a check on the accuracy of the solution. The other information contains a (discretized) check on the necessary conditions of optimality described in section 2 - the extremes of the inequalities are listed together with the distances (and ground values) where they occurred (only internal grid points of the sections are considered in searching for these extremes). In the output; CVx refers to the co-variable λ_1 and $x = 1,3,5$ denotes sections A, C, B, respectively; for optimality, the first three values listed should be positive and the last negative.

(ii) The level 2 output (a file for spooling) contains the same information as the level 1 output (but for the last optimality information which is encoded differently). Furthermore, a full table of results containing all grid points and values of speeds, distances (or co-variables λ_1, λ_2) and ground function g for the sections A, C and B is included for every problem. The times are not printed out but can be recovered as follows: denoting $t_0 = 0$, $t_4 = T$ and t_1 , t_2 , t_3 the switching times when the sections A, M and C end, respectively, then the time t corresponding to a grid point s (note that $0 \le s \le 1$) is

$$t = t_k + s(t_{k+1} - t_k)$$

where $k = 0,2,3$ for the sections A, C, B, respectively. Note that the same applies for the M section (k = 1) which, however, having constant speed (and co-variable λ_1) is not documented.

The level 2 output also contains parameter lists before each AB, AMCB or standard RUN problem (it is therefore advisable to do the AC problem only after the AB problem). Furthermore, an annotation of the internal plot is included for easy identification (see later).

(iii) The level 3 output (a file for spooling) contains the tables of initial guesses and solutions of all problems solved by PASVAR. These include the switching times as "constant" functions. The annotation is minimal, the columns contain grid points, speed and distance (and or λ_1 and λ_2) for all sections (but M) and, last, the switching times. The parameter lists are included as in level 2 output.

(iv) The internal plot is designed to provide quick graphic information about the solution on a (suitable) terminal. The software used is a general plotting package developed at the Flinders University which allows the direction of the output to various media as described in section 4(e). The design used was influenced by the advantages and limitations of the GIGI graphic terminal.

Four graphs are produced: speed, co-variable λ_1, ground function and the hill function - in that order - as functions of time. The graphs are easy to identify: speed is positive starting and finishing at zero level, λ_1 (same scale as speed) intersects speed at the end of A section and is zero at the beginning of the B section, hill function gives the shape of the hill starting at zero while the ground function is its derivative (slope, with respect to distance). No characters are plotted to speed up the process; however, entering any character (say •) and carriage return *after* the beep indicating the end of plotting will display an identification containing the name of the last data input file ("MANUAL" if none) and the values of axis increments for the time (horizontal axis, equal to $0.1 * T$), speed, ground and hill functions (vertical axis, all enlarged to ensure readability). At the bottom the graph contains a saw-like line indicating the positions of the grid points used by PASVAR in solving the problem. Their density helps to identify the separation of the A, C and B sections; however the flat level in the M section is used to indicate the zero level if the ground function is interpreted as the control during that section. In particular, should the ground function intersect this flat segment, such AMCB solution will not be optimal as, to maintain the speed required, the control will require braking and another, more complex, strategy will be called for.

(v) The external plot uses the DD80 plotting system available at Flinders University. It produces (on a printer-plotter) the list of case parameters for identification and two plots of functions depending on the *distance*. The first plots contains three functions related to the accelaration scale; $p(v) = a + bv + cv^2$, ground $g(x)$ and control u where $v = \dot{x}$ is the speed (again as a function of distance x). The second plot contains the speed related functions $v = \dot{x}$ and λ_1 and the hill function (accumulated ground). The scale factor between the hill function and speed is quoted (this is chosen automatically but user is given the option to update it during the production of the graphs).

(vi) The PASVAR related output is self-explanatory.

6. Some implementation comments.

The PASVAR3 used in this application was the PFORT verified HARWELL library type package (main subroutine called DD04AD) of approximately 1800 FORTRAN statements (including comments).

The OTEP-2 is written in FORTRAN77 and was developed on the Flinders University PR1ME-850 system using the University of Salford FTN77 compiler; it has about 2300 statements and comments.

The data space requirement of the OTEP itself are minimal (∿2000 locations).The PASVAR space is organized in two working fields (integer and floating points) sizes of which limit the number of grid points PASVAR may use. Currently, these are set to 16000 (easy to change as compilation parameters LWORK and LIWORK); this choice allows each of these fields to fit into a separate segment - the compilation switch BIG was thus not necessary.

Of the non-standard features used we mention the SUBROUTINE TNOUA (S,L) and FUNCTION INTS (I). Essentially, TNOUA is a PR1ME system library procedure allowing the display of a prompt at a terminal without advancing to the next line for the following input - a useful feature for economizing the limited display of menus. Here L is a short integer (2 bytes) giving the length of the character constant S , INTS converts the integer accordingly. Example of use:

```
CALL TNOUA ('KEY =', INTS(5))
READ (LTI,*) ISW
```

will prompt and obtain the value of an integer ISW using one line of the terminal display only.

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Appendix A .

List of menus to input data and to control OTEP - 2 .

MENU: MAIN
****

KEY	RESPONSE
1	RUN
2	STOP
3	CASE PARAMETERS
4	PASVAR PARAMETERS
5	I/O CONTROL PARAMETERS
6	PLOTING OPTION (EXTERNAL)
7	AB
8	AC
9	AVB
10	AVC
11	ACB
12	AVCB
13	AMCB

KEY=

```
MENU:  CASE PARAMETERS
*********************
KEY            PARAMETER            CURRENT VALUE
===            =========            =============

 1             TIME                 0.2000D+01
 2             DISTANCE             0.2000D+01
 3             U(ACC.)              0.1000D+02
 4             U(BR.)               -.2000D+01
 5             B(ACC.)              -.1000D-01
 6             C(ACC.)              -.1000D-01
 7             B(BR.)               -.1000D-01
 8             C(BR.)               -.1000D-01
 9             A                    0.3000D-00
10             B                    0.1400D-00
11             C                    0.1600D-00
12             VMAX                 0.1000D+01
13             MASS                 0.1000D+01
14             GROUND               LEVEL
15          ** MENU:  MAIN **
16          ** RUN **

KEY=
```

```
MENU:  GROUND PARAMETERS
***********************


KEY            PARAMETER            CURRENT VALUE
===            =========            ============

 1             EPSILON              0.1000D+01
 2             NO. OF B.P.'S        1

                                    POSITION      GROUND
                                    ========      ======
 3             B.P.# 1              0.2000D+01    0.0000D-00
 4          ** MENU; MAIN **
 5          ** MENU:CASE PARAMETERS **
 6          ** RUN **

KEY=
```

```
MENU:  PASVAR  PARAMETERS
*************************


KEY            PARAMETER                                CURRENT VALUE
===            =========                                =============

 1             INITIAL NO.  OF GRID POINTS (ACB)         11
 2             MAXIMUM NO.  OF GRID POINTS (ACB)         21
 3             MAXIMUM NO.  OF GRID POINTS (AMCB)        33
 4             TOLERANCE                                 0.1D-04
 5             ** MENU:  MAIN **
 6             ** RUN **


KEY=
```

```
MENU:  I/O PARAMETERS
*********************
KEY            PARAMETER                       CURRENT VALUE
===            =========                       =============

 1             RUN
 2             STOP
 3             MAIN MENU
 4             PRINT CONTROL                         2
 5             INTERACTIVE INPUT DEVICE NO.          1
 6             INTERACTIVE OUTPUT DEVICE NO.         1
 7             LEVEL 2 DEVICE NO.                   12
 8             LEVEL 3 DEVICE NO.                   13
 9             ERROR MESSAGE DEVICE NO.             14
 10            PROGRESS INFO DEVICE NO.              0
 11            PLOT CONTROL                          0


KEY=
```

Appendix B.

Sample input file .

```
6             to disconnect the terminal output    (simple ground,
9        it will go on a scratch file                  smooth)
3             back to MAIN menu
****          respond to  "beep"
3             requests CASE menu
1            change time
2.3            new time
12           change  VMAX
1.6            new VMAX
14             new GROUND to be entered
2             change number of
3                breaking points
.5,1.0               first
2.,-0.5               second
2.5,1.0               third
1             change
.5                 epsilon (smoothing parameter)
6             back to MAIN menu ( 3+No. of br. pnts. !)
****           respond to  "beep"
5             to I/O menu
4           change print control to
3           level 3
6             reset the terminal as
1                 output device
5                 and input
1                 device
```

Appendix C.

Sample terminal (level 1) output.

The case run is that given by the sample input file from Appendix B. The parameter values used do not represent any real situation.

The corresponding level 2, level 3 and PASVAR diagnostics outputs are in Appendices D, E and F.

```
MENU:  MAIN
**********


KEY           RESPONSE
===           ========

 1            RUN
 2            STOP
 3            CASE PARAMETERS
 4            PASVAR PARAMETERS
 5            I/O CONTROL PARAMETERS
 6            PLOTING OPTION (EXTERNAL)
 7            AB
 8            AC
 9            AVB
 10           AVC
 11           ACB
 12           AVCB
 13           AMCB

KEY= 5                  to I/O menu



MENU:  I/O PARAMETERS
********************
KEY           PARAMETER                          CURRENT VALUE
===           =========                          =============

 1            RUN
 2            STOP
 3            MAIN MENU
 4            PRINT CONTROL                             2
 5            INTERACTIVE INPUT DEVICE NO.              1
 6            INTERACTIVE OUTPUT DEVICE NO.             1
 7            LEVEL 2 DEVICE NO.                       12
 8            LEVEL 3 DEVICE NO.                       13
 9            ERROR MESSAGE DEVICE NO.                 14
 10           PROGRESS INFO DEVICE NO.                  0
 11           PLOT CONTROL                              0

KEY= 11                        to change plot control
NEW PLOT CONTROL= -4           GIGI plots
KEY= 5                         to input data from a file
NEW DEVICE NO. = 10
KEY= NEW DEVICE NO. =   ...    prompts from S/R TNOUA follow until ....
KEY=  1    to RUN the standard steps automatically
```

```
==========================================================================
     ACB STRATEGY - PASVAR SOLN'S      ACB STRATEGY - PASVAR SOLN'S
==========================================================================

***** PASVAR PERFORMANCE      PASVAR ERROR MESSAGE=  0
                              NO.  OF GRID POINTS=   19
                              MAX PASVAR ERROR=      0.7097D-05
                              PASVAR TIME=           0.7233D+01

***** RESULTS

EVENT           SWITCH TIME        DISTANCE      MAX. SPEED         ENERGY
-----           -----------        --------      ----------         ------
A (ENDS)            0.21365         0.19796        1.81028         1.92227
B (STARTS)          2.23415         1.99412        0.17885            FREE

***** DECISION (KEY)      *** CONTINUE (0) *** MAIN MENU (1) ***
***** KEY= 0           solution successful




==========================================================================
     ACB STRATEGY - OPTIMALITY         ACB STRATEGY - OPTIMALITY
==========================================================================

***** PASVAR PERFORMANCE      PASVAR ERROR MESSAGE=  2
                              NO.  OF GRID POINTS=   19
                              MAX PASVAR ERROR=      0.2741D-04
                              PASVAR TIME=           0.1664D+01

***** RESULTS

 CV(4) - DPOL(Y3(1)) * Y3(1)       =-.1138D+01
       + POL(Y3(1)) + G(Y4(0)

THEREFORE THE ACB STRATEGY IS NOT OPTIMAL

                        DISTANCE     GROUND      VALUE
                        --------     ------      -----
MIN(CV1 - SPEED)         0.07244    0.70897   -0.04241
MIN(CV3)                 1.97234    0.37992    0.16530
MIN(SPEED - CV3)         1.27192    0.06176   -0.39777
MAX(CV5)                 1.99432    0.39874   -0.00182

        GIGI  plotting information follows here ...
OTIN18  STEPS:  TIME 0.2300 ,  SPEED 0.3017 ,  GROUND 0.1649 ,  HILL 0.1238
```

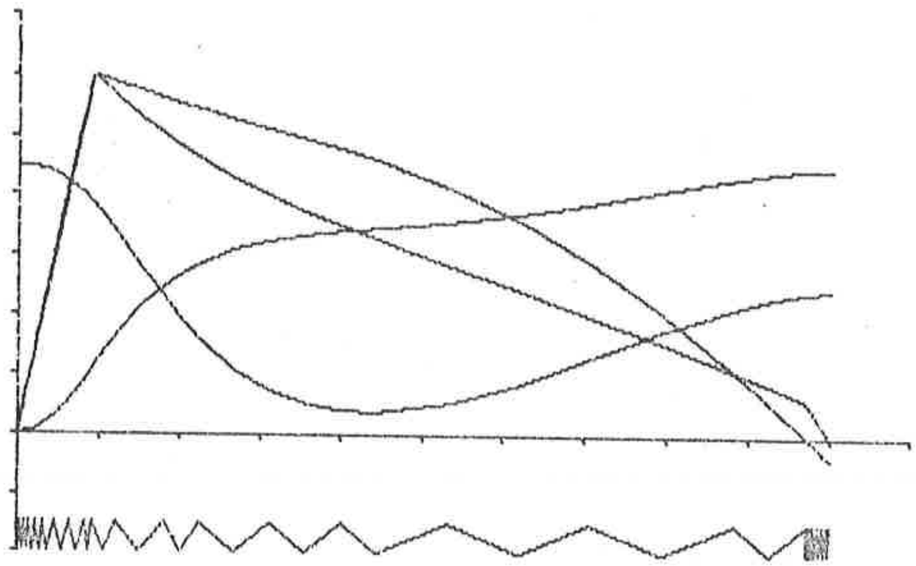
```
=================================================================================
    AMCB STRATEGY - PASVAR SOLN'S     AMCB STRATEGY - PASVAR SOLN'S
=================================================================================

***** PASVAR PERFORMANCE       PASVAR ERROR MESSAGE=  0
                               NO. OF GRID POINTS=   19
                               MAX PASVAR ERROR=     0.1714D-05
                               PASVAR TIME=          0.1359D+02

***** RESULTS

EVENT            SWITCH TIME       DISTANCE      MAX. SPEED          ENERGY
-----            -----------       --------      ----------          ------
A (ENDS)             0.12701        0.07113         1.10832         0.70165
M (ENDS)             1.14224        1.19633         ===%===         0.73329 (VEL.)
                                                                    0.37190 (GRND.
  MAX(CTR.)          1.39388        0.07113                      ==========
  MIN(CTR.)          0.73187        1.19633                         1.80684 (TOT.)
                                                                 ==========
B (STARTS)           2.14911        1.96900         0.41232            FREE

***** DECISION (KEY)      *** CONTINUE (0) *** MAIN MENU (1) ***
***** KEY= 0



=================================================================================
    AMCB STRATEGY - OPTIMALITY        AMCB STRATEGY - OPTIMALITY
=================================================================================

***** PASVAR PERFORMANCE       PASVAR ERROR MESSAGE=  0
                               NO. OF GRID POINTS=   19
                               MAX PASVAR ERROR=     0.1531D-05
                               PASVAR TIME=          0.1324D+01

***** RESULTS

 CV(4) - DPOL(Y3(1)) * Y3(1)      =0.1862D-08
       + POL(Y3(1)) + G(Y4(0))

                      DISTANCE     GROUND      VALUE
                      --------     ------      -----
MIN(CV1 - SPEED)       0.05774    0.71608   -0.00140
MIN(CV3)               1.94728    0.35867    0.07364
MIN(SPEED - CV3)       1.21483    0.06217    0.00013
MAX(CV5)               1.97003    0.37795   -0.00371

    GIGI plotting information follows ...
OTIN18  STEPS:  TIME 0.2300 ,  SPEED 0.1847 ,  GROUND 0.1649 ,  HILL 0.1238
```

```
MENU:  MAIN
**********

KEY             RESPONSE
===             ========

 1              RUN
 2              STOP
 3              CASE PARAMETERS
 4              PASVAR PARAMETERS
 5              I/O CONTROL PARAMETERS
 6              PLOTING OPTION (EXTERNAL)
 7              AB
 8              AC
 9              AVB
 10             AVC
 11             ACB
 12             AVCB
 13             AMCB

KEY= 12                        to solve the speed restricted problem


================================================================================
    AVCB STRATEGY - PASVAR SOLN'S     AVCB STRATEGY - PASVAR SOLN'S
================================================================================

***** PASVAR PERFORMANCE       PASVAR ERROR MESSAGE=  0
                               NO. OF GRID POINTS=    18
                               MAX PASVAR ERROR=      0.4005D-05
                               PASVAR TIME=           0.7382D+01

***** RESULTS

EVENT          SWITCH TIME       DISTANCE       MAX. SPEED        ENERGY
-----          -----------       --------       ----------        ------
A (ENDS)         0.18697          0.15245        1.60000        1.48819
V (ENDS)         0.36038          0.42990        ===%===        0.25903 (VEL.)
                                                                0.15856 (GRND.
  MAX(CTR.)      1.66926          0.15245                     ==========
  MIN(CTR.)      1.45796          0.42990                       1.90578 (TOT.)
                                                              ==========
B (STARTS)       2.23047          1.99344        0.18891          FREE

    GIGI plotting information follows
OTIN18  STEPS:  TIME 0.2300 ,  SPEED 0.2667 ,  GROUND 0.1649 ,  HILL 0.1238
```

```
MENU: MAIN
**********

KEY          RESPONSE
===          ========

 1           RUN
 2           STOP
 3           CASE PARAMETERS
 4           PASVAR PARAMETERS
 5           I/O CONTROL PARAMETERS
 6           PLOTING OPTION (EXTERNAL)
 7           AB
 8           AC
 9           AVB
10           AVC
11           ACB
12           AVCB
13           AMCB

KEY= 2       to exit
**** STOP
```

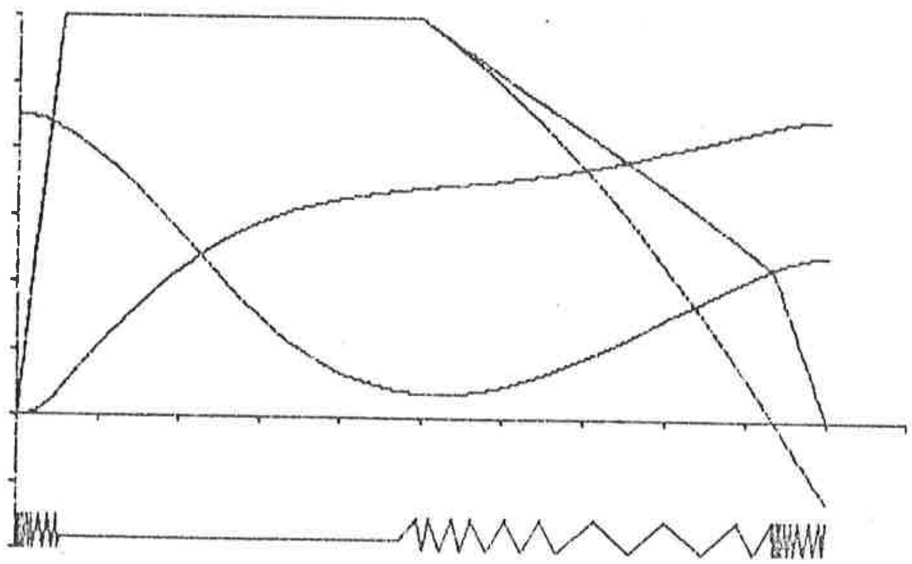
Remark: The example solved here involves an uphill ground of varying steepness (see the GIGI plots on the following two pages). The results of the step 2(ii) - AMCB optimality indicate that $\lambda_1 < \dot{x}$ on the A section. Noting the smallness of the difference $|\dot{x} - \lambda_1|$ we suspect that this is due, in this case, to the approximate nature of the solution rather than to the AMCB strategy not being optimal.

The ACB solution.



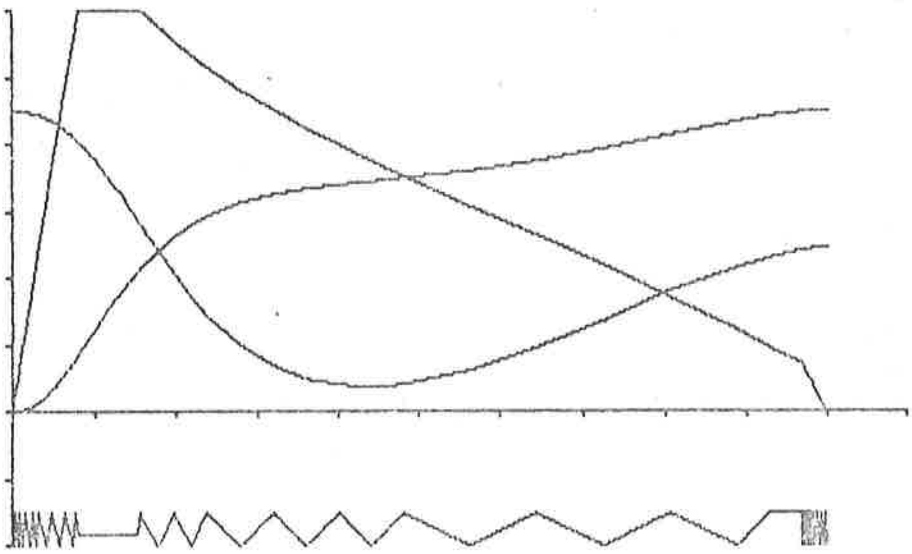
OTIN18 STEPS: TIME 0.2300 , SPEED 0.3017 , GROUND 0.1649 , HILL 0.1238

The AMCB solution.



OTIN18 STEPS: TIME 0.2300 , SPEED 0.1847 , GROUND 0.1649 , HILL 0.1238

The AVCB solution.



OTIN18 STEPS: TIME 0.2300 , SPEED 0.2667 , GROUND 0.1649 , HILL 0.1238

Appendix D.

Sample level 2 output.

```
******************
* PARAMETER LIST *
******************
```

CASE PARAMETERS

KEY	PARAMETER	CURRENT VALUE
1	TIME	0.2300D+01
2	DISTANCE	0.2000D+01
3	U(ACC.)	0.1000D+02
4	U(BR.)	-.2000D+01
5	B(ACC.)	-.1000D-01
6	C(ACC.)	-.1000D-01
7	B(BR.)	-.1000D-01
8	C(BR.)	-.1000D-01
9	A	0.3000D-00
10	B	0.1400D-00
11	C	0.1600D-00
12	VMAX	0.1600D+01
13	MASS	0.1000D+01

GROUND PARAMETERS

KEY	PARAMETER	CURRENT VALUE	
1	EPSILON	0.5000D-00	
2	NO. OF B.P.'S	3	
		POSITION	GROUND
3	B.P.# 1	0.5000D-00	0.1000D+01
4	B.P.# 2	0.2000D+01	-.5000D-00
5	B.P.# 3	0.2500D+01	0.1000D+01

PASVAR PARAMETERS

KEY	PARAMETER	CURRENT VALUE
1	INITIAL NO. OF GRID POINTS (ACB)	11
2	MAXIMUM NO. OF GRID POINTS (ACB)	21
3	MAXIMUM NO. OF GRID POINTS (AMCB)	33
4	TOLERANCE	0.1D-04

ACB STRATEGY - PASVAR SOLN'S   ACB STRATEGY - PASVAR SOLN'S   ACB STRATEGY - PASVAR SOLN'S   ACB STRATEGY - PASVAR SOLN'S

***** PASVAR PERFORMANCE

PASVAR ERROR MESSAGE= 0
NO. OF GRID POINTS= 19
MAX PASVAR ERROR= 0.7097D-05
PASVAR TIME= 0.7233D+01

***** RESULTS

EVENT	SWITCH TIME	DISTANCE	MAX. SPEED	ENERGY
A (ENDS)	0.21365	0.19796	1.81028	1.92227
B (STARTS)	2.23415	1.99412	0.17885	FREE

GRID PT'S	ACCELERATION PHASE DISTANCE	ACCELERATION PHASE GROUND	ACCELERATION PHASE SPEED	COASTING PHASE DISTANCE	COASTING PHASE GROUND	COASTING PHASE SPEED	BRAKING PHASE DISTANCE	BRAKING PHASE GROUND	BRAKING PHASE SPEED
0.00000	0.00000	0.74197	0.00000	0.19796	0.63869	1.81028	1.99412	0.39857	0.17885
0.01667	0.00006	0.74194	0.03188	0.25796	0.59871	1.75383	1.99432	0.39874	0.17586
0.03333	0.00023	0.74187	0.06374	0.31611	0.55609	1.70007	1.99451	0.39891	0.17286
0.06667	0.00091	0.74159	0.12736	0.42720	0.46600	1.60055	1.99488	0.39923	0.16683
0.10000	0.00204	0.74111	0.19084	0.53193	0.37628	1.51127	1.99524	0.39953	0.16089
0.12500	0.00319	0.74062	0.23835	0.60672	0.31411	1.45052	1.99550	0.39976	0.15640
0.15000	0.00459	0.74003	0.28578	0.67855	0.25895	1.39452	1.99575	0.39998	0.15192
0.20000	0.00815	0.73851	0.38034	0.81426	0.17262	1.29450	1.99624	0.40039	0.14295
0.25000	0.01271	0.73654	0.47447	0.94052	0.11569	1.20692	1.99670	0.40079	0.13398
0.30000	0.01828	0.73411	0.56814	1.05842	0.08175	1.12828	1.99712	0.40115	0.12502
0.35000	0.02485	0.73122	0.66131	1.16871	0.06510	1.05597	1.99752	0.40149	0.11607
0.40000	0.03241	0.72783	0.75393	1.27192	0.06176	0.98803	1.99789	0.40181	0.10711
0.50000	0.05048	0.71951	0.93738	1.45845	0.08555	0.85942	1.99853	0.40236	0.08922
0.60000	0.07244	0.70897	1.11820	1.61938	0.13991	0.73337	1.99906	0.40282	0.07134
0.70000	0.09824	0.69594	1.29613	1.75457	0.21321	0.60383	1.99947	0.40317	0.05348
0.80000	0.12780	0.68014	1.47092	1.86294	0.29014	0.46777	1.99977	0.40342	0.03564
0.90000	0.16107	0.66119	1.64236	1.94315	0.35519	0.32527	1.99994	0.40357	0.01781
0.95000	0.17907	0.65041	1.72677	1.97234	0.37992	0.25234	1.99999	0.40361	0.00890
1.00000	0.19796	0.63869	1.81028	1.99412	0.37857	0.17885	2.00000	0.40362	0.00000

```
***** PASVAR PERFORMANCE      PASVAR ERROR MESSAGE=  2
                              NO. OF GRID POINTS=    19
                              MAX PASVAR ERROR=      0.2741D-04
                              PASVAR TIME=           0.1664D+01

***** RESULTS

 CV(4) - DPOL(Y3(1)) * Y3(1)       =-.1138D+01
     + POL(Y3(1)) + G(Y4(0))

THEREFORE THE ACB STRATEGY IS NOT OPTIMAL
```

	ACCELERATION PHASE			COASTING PHASE				BRAKING PHASE		
GRID P'TS	DISTANCE	GROUND	CV1-SPEED	DISTANCE	GROUND	CV3	SPEED-CV3	DISTANCE	GROUND	-CV5
0.00000	0.00000	0.74197	0.03307	0.19796	0.63869	1.81028	0.00000	1.99412	0.39857	0.00000
0.01667	0.00006	0.74194	0.02977	0.25796	0.59871	1.79070	-0.03687	1.99432	0.39874	0.00182<
0.03333	0.00023	0.74187	0.02650	0.31611	0.55609	1.77101	-0.07094	1.99451	0.39891	0.00364
0.06667	0.00091	0.74159	0.02009	0.42720	0.46600	1.73203	-0.13149	1.99488	0.39923	0.00727
0.10000	0.00204	0.74111	0.01384	0.53193	0.37628	1.69445	-0.18317	1.99524	0.39953	0.01091
0.12500	0.00319	0.74062	0.00929	0.60672	0.31411	1.66748	-0.21696	1.99550	0.39976	0.01364
0.15000	0.00459	0.74003	0.00486	0.67855	0.25895	1.64155	-0.24703	1.99575	0.39998	0.01637
0.20000	0.00815	0.73851	-0.00360	0.81426	0.17262	1.59201	-0.29750	1.99624	0.40039	0.02184
0.25000	0.01271	0.73654	-0.01145	0.94052	0.11569	1.54374	-0.33682	1.99670	0.40079	0.02730
0.30000	0.01828	0.73411	-0.01861	1.05842	0.08175	1.49452	-0.36624	1.99712	0.40115	0.03277
0.35000	0.02485	0.73122	-0.02499	1.16871	0.06510	1.44243	-0.38646	1.99752	0.40149	0.03824
0.40000	0.03241	0.72763	-0.03052	1.27192	0.06176	1.38580	-0.39777<	1.99789	0.40181	0.04371
0.50000	0.05048	0.71951	-0.03867	1.45845	0.08555	1.25305	-0.39363	1.99853	0.40236	0.05466
0.60000	0.07244	0.70897	-0.04241<	1.61938	0.13991	1.08579	-0.35242	1.99906	0.40282	0.06562
0.70000	0.09824	0.69594	-0.04108	1.75457	0.21321	0.87608	-0.27225	1.99947	0.40317	0.07658
0.80000	0.12780	0.68014	-0.03402	1.86294	0.29014	0.62104	-0.15327	1.99977	0.40342	0.08755
0.90000	0.16107	0.66119	-0.02055	1.94315	0.35519	0.32515	0.00012	1.99994	0.40357	0.09852
0.95000	0.17907	0.65041	-0.01120	1.97234	0.37992	0.16530<	0.08704	1.99999	0.40361	0.10400
1.00000	0.19796	0.63869	0.00000	1.99412	0.39857	0.00000	0.17885	2.00000	0.40362	0.10949

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

OTIN18 STEPS: TIME 0.2300 , SPEED 0.3017 , GROUND 0.1649 , HILL 0.1238

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

AMCB STRATEGY - PASVAR SOLN'S AMCB STRATEGY - PASVAR SOLN'S AMCB STRATEGY - PASVAR SOLN'S AMCB STRATEGY - PASVAR SOLN'S

***** PASVAR PERFORMANCE

PASVAR ERROR MESSAGE=	0
NO. OF GRID POINTS=	19
MAX PASVAR ERROR=	0.1714D-05
PASVAR TIME=	0.1359D+02

***** RESULTS

EVENT	SWITCH TIME	DISTANCE	MAX. SPEED	ENERGY	
A (ENDS)	0.12701	0.07113	1.10832	0.70165	
M (ENDS)	1.14224	1.19633	===%===	0.73329	(VEL.)
				0.37190	(GRND.)
MAX(CTR.)	1.39388	0.07113		==========	
MIN(CTR.)	0.73187	1.19633		1.80684	(TOT.)
				==========	
B (STARTS)	2.14911	1.96900	0.41232	FREE	

GRID PT'S	ACCELERATION PHASE DISTANCE	ACCELERATION PHASE GROUND	ACCELERATION PHASE SPEED	COASTING PHASE DISTANCE	COASTING PHASE GROUND	COASTING PHASE SPEED	BRAKING PHASE DISTANCE	BRAKING PHASE GROUND	BRAKING PHASE SPEED
0.00000	0.00000	0.74197	0.00000	1.19633	0.06307	1.10832	1.96900	0.37708	0.41232
0.01667	0.00002	0.74196	0.01896	1.21483	0.06217	1.09638	1.97003	0.37795	0.40537
0.03333	0.00008	0.74193	0.03791	1.23313	0.06166	1.08455	1.97104	0.37882	0.39843
0.06667	0.00032	0.74183	0.07577	1.26913	0.06170	1.06120	1.97301	0.38050	0.38455
0.10000	0.00072	0.74166	0.11359	1.30436	0.06311	1.03819	1.97491	0.38212	0.37069
0.12500	0.00113	0.74149	0.14193	1.33028	0.06501	1.02113	1.97629	0.38330	0.36029
0.15000	0.00162	0.74128	0.17024	1.35577	0.06760	1.00421	1.97763	0.38444	0.34991
0.20000	0.00288	0.74075	0.22676	1.40548	0.07476	0.97072	1.98019	0.38663	0.32915
0.25000	0.00450	0.74006	0.28317	1.45352	0.08440	0.93756	1.98260	0.38869	0.30841
0.30000	0.00648	0.73922	0.33944	1.49989	0.09634	0.90459	1.98484	0.39062	0.28770
0.35000	0.00881	0.73822	0.39556	1.54460	0.11042	0.87168	1.98674	0.39241	0.26700
0.40000	0.01150	0.73706	0.45154	1.58765	0.12647	0.83870	1.98887	0.39407	0.24633
0.50000	0.01795	0.73426	0.56300	1.66875	0.16364	0.77213	1.99228	0.39699	0.20506
0.60000	0.02580	0.73079	0.67375	1.74309	0.20597	0.70416	1.99506	0.39938	0.16387
0.70000	0.03506	0.72663	0.78372	1.81049	0.25105	0.63429	1.99722	0.40124	0.12277
0.80000	0.04571	0.72174	0.89284	1.87073	0.29622	0.56231	1.99877	0.40257	0.08176
0.90000	0.05774	0.71608	1.00106	1.92365	0.33892	0.48824	1.99969	0.40336	0.04084
0.95000	0.06426	0.71295	1.05481	1.94728	0.35867	0.45049	1.99992	0.40356	0.02041
1.00000	0.07113	0.70961	1.10832	1.96900	0.37708	0.41232	2.00000	0.40362	0.00000

***** PASVAR PERFORMANCE

PASVAR ERROR MESSAGE= 0
NO. OF GRID POINTS= 19
MAX PASVAR ERROR= 0.1531D-05
PASVAR TIME= 0.1324D+01

***** RESULTS

CV(4) - DPOL(Y3(1)) * Y3(1) + POL(Y3(1)) + G(Y4(O)) =0.1862D-08

	ACCELERATION PHASE			COASTING PHASE				BRAKING PHASE		
GRID P'TS	DISTANCE	GROUND	CV1-SPEED	DISTANCE	GROUND	CV3	SPEED-CV3	DISTANCE	GROUND	-CV5
0.00000	0.00000	0.74197	0.04548	1.19633	0.06307	1.10832	0.00000	1.96900	0.37708	0.00000
0.01667	0.00002	0.74196	0.04398	1.21483	0.06217	1.09624	0.00013<	1.97003	0.37795	0.00371<
0.03333	0.00008	0.74193	0.04250	1.23313	0.06166	1.08401	0.00054	1.97104	0.37882	0.00742
0.06667	0.00032	0.74183	0.03957	1.26913	0.06170	1.05906	0.00214	1.97301	0.38050	0.01484
0.10000	0.00072	0.74166	0.03670	1.30436	0.06311	1.03341	0.00478	1.97491	0.38212	0.02227
0.12500	0.00113	0.74149	0.03459	1.33028	0.06501	1.01369	0.00744	1.97629	0.38330	0.02785
0.15000	0.00162	0.74128	0.03251	1.35577	0.06760	0.99354	0.01067	1.97763	0.38444	0.03344
0.20000	0.00288	0.74075	0.02848	1.40548	0.07476	0.95190	0.01882	1.98019	0.38663	0.04462
0.25000	0.00450	0.74006	0.02463	1.45352	0.08440	0.90836	0.02920	1.98260	0.38869	0.05581
0.30000	0.00648	0.73922	0.02097	1.49989	0.09634	0.86283	0.04176	1.98484	0.39062	0.06701
0.35000	0.00881	0.73822	0.01752	1.54460	0.11042	0.81521	0.05647	1.98694	0.39241	0.07823
0.40000	0.01150	0.73706	0.01430	1.58765	0.12647	0.76542	0.07328	1.98887	0.39407	0.08945
0.50000	0.01795	0.73426	0.00861	1.66875	0.16364	0.65911	0.11302	1.99228	0.39699	0.11192
0.60000	0.02580	0.73079	0.00406	1.74309	0.20597	0.54363	0.16053	1.99506	0.39938	0.13441
0.70000	0.03506	0.72663	0.00078	1.81049	0.25105	0.41914	0.21516	1.99722	0.40124	0.15691
0.80000	0.04571	0.72174	-0.00109	1.87075	0.29622	0.28625	0.27607	1.99877	0.40257	0.17940
0.90000	0.05774	0.71608	-0.00140<	1.92365	0.33892	0.14604	0.34220	1.99969	0.40336	0.20187
0.95000	0.06426	0.71295	-0.00092	1.94728	0.35867	0.07364<	0.37685	1.99992	0.40356	0.21309
1.00000	0.07113	0.70961	0.00000	1.96900	0.37708	0.00000	0.41232	2.00000	0.40362	0.22431

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

CTIN18 STEPS: TIME 0.2300 , SPEED 0.1847 , GROUND 0.1649 , HILL 0.1238

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

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******************
* PARAMETER LIST *
******************


CASE PARAMETERS
***************
KEY            PARAMETER            CURRENT VALUE
===            =========            =============

 1             TIME                 0. 2300D+01
 2             DISTANCE             0. 2000D+01
 3             U(ACC. )             0. 1000D+02
 4             U(BR. )              -. 2000D+01
 5             B(ACC. )             -. 1000D-01
 6             C(ACC. )             -. 1000D-01
 7             B(BR. )              -. 1000D-01
 8             C(BR. )              -. 1000D-01
 9             A                    0. 3000D-00
 10            B                    0. 1400D-00
 11            C                    0. 1600D-00
 12            VMAX                 0. 1600D+01
 13            MASS                 0. 1000D+01

GROUND PARAMETERS
*****************

KEY            PARAMETER            CURRENT VALUE
===            =========            =============

 1             EPSILON              0. 5000D-00
 2             NO. OF B. P. 'S      3

                                    POSITION     GROUND
                                    ========     ======
 3             B. P. # 1            0. 5000D-00  0. 1000D+01
 4             B. P. # 2            0. 2000D+01  -. 5000D-00
 5             B. P. # 3            0. 2500D+01  0. 1000D+01

PASVAR PARAMETERS
*****************

KEY            PARAMETER                                  CURRENT VALUE
===            =========                                  =============

 1             INITIAL NO. OF GRID POINTS (ACB)            11
 2             MAXIMUM NO. OF GRID POINTS (ACB)            21
 3             MAXIMUM NO. OF GRID POINTS (AMCB)           33
 4             TOLERANCE                                   0. 1D-04
```

***** PASVAR PERFORMANCE

PASVAR ERROR MESSAGE= 0
NO. OF GRID POINTS= 18
MAX PASVAR ERROR= 0.4005D-05
PASVAR TIME= 0.7382D+01

***** RESULTS

EVENT	SWITCH TIME	DISTANCE	MAX. SPEED	ENERGY
A (ENDS)	0.18697	0.15245	1.60000	1.48819
V (ENDS)	0.36038	0.42990	===%===	0.25903 (VEL.)
				0.15856 (GRND.)
MAX(CTR.)	1.66926	0.15245		
MIN(CTR.)	1.45796	0.42990		1.90578 (TOT.)
B (STARTS)	2.23047	1.99344	0.18891	FREE

GRID PT'S	ACCELERATION PHASE DISTANCE	ACCELERATION PHASE GROUND	ACCELERATION PHASE SPEED	COASTING PHASE DISTANCE	COASTING PHASE GROUND	COASTING PHASE SPEED	BRAKING PHASE DISTANCE	BRAKING PHASE GROUND	BRAKING PHASE SPEED
0.00000	0.00000	0.74197	0.00000	0.42990	0.46371	1.60000	1.99344	0.39799	0.18891
0.02500	0.00010	0.74193	0.04185	0.50321	0.40086	1.53711	1.99377	0.39827	0.18417
0.05000	0.00039	0.74180	0.08364	0.57370	0.34111	1.47888	1.99408	0.39854	0.17942
0.07500	0.00088	0.74160	0.12539	0.64156	0.28666	1.42494	1.99439	0.39881	0.17468
0.10000	0.00156	0.74131	0.16707	0.70700	0.23877	1.37485	1.99469	0.39906	0.16994
0.15000	0.00351	0.74048	0.25025	0.83123	0.16365	1.28438	1.99526	0.39956	0.16046
0.20000	0.00624	0.73932	0.33315	0.94751	0.11318	1.20414	1.99581	0.40002	0.15098
0.25000	0.00974	0.73782	0.41574	1.05665	0.08213	1.13142	1.99631	0.40046	0.14151
0.30000	0.01401	0.73598	0.49799	1.15926	0.06598	1.06415	1.99679	0.40087	0.13205
0.35000	0.01905	0.73378	0.57989	1.25578	0.06152	1.00077	1.99723	0.40125	0.12258
0.40000	0.02485	0.73121	0.66139	1.34649	0.06657	0.94001	1.99764	0.40160	0.11313
0.50000	0.03873	0.72496	0.82311	1.51125	0.09968	0.82247	1.99836	0.40222	0.09423
0.60000	0.05562	0.71709	0.98296	1.65413	0.15625	0.70522	1.99895	0.40272	0.07534
0.70000	0.07548	0.70747	1.14073	1.77476	0.22639	0.58408	1.99941	0.40312	0.05648
0.80000	0.09826	0.69593	1.29627	1.87221	0.29737	0.45711	1.99974	0.40340	0.03764
0.90000	0.12393	0.68226	1.44941	1.94539	0.35707	0.32469	1.99993	0.40357	0.01881
0.95000	0.13784	0.67455	1.52503	1.97259	0.38014	0.25706	1.99998	0.40361	0.00940
1.00000	0.15245	0.66622	1.60000	1.99344	0.39799	0.18891	2.00000	0.40362	0.00000

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

OTIN18 STEPS: TIME 0.2300 , SPEED 0.2667 , GROUND 0.1649 , HILL 0.1238

GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO GRAPH INFO

Appendix E .

Sample level 3 output.

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*****************
* PARAMETER LIST *
*****************


CASE PARAMETERS
***************
KEY            PARAMETER           CURRENT VALUE
===            =========           =============

 1             TIME                0. 2300D+01
 2             DISTANCE            0. 2000D+01
 3             U(ACC. )            0. 1000D+02
 4             U(BR. )             -. 2000D+01
 5             B(ACC. )            -. 1000D-01
 6             C(ACC. )            -. 1000D-01
 7             B(BR. )             -. 1000D-01
 8             C(BR. )             -. 1000D-01
 9             A                   0. 3000D-00
 10            B                   0. 1400D-00
 11            C                   0. 1600D-00
 12            VMAX                0. 1600D+01
 13            MASS                0. 1000D+01

GROUND PARAMETERS
*****************

KEY            PARAMETER           CURRENT VALUE
===            =========           =============

 1             EPSILON             0. 5000D-00
 2             NO.  OF B. P. 'S    3

                                   POSITION     GROUND
                                   ========     ======
 3             B. P. # 1           0. 5000D-00  0. 1000D+01
 4             B. P. # 2           0. 2000D+01  -. 5000D-00
 5             B. P. # 3           0. 2500D+01  0. 1000D+01

PASVAR PARAMETERS
*****************

KEY            PARAMETER                                 CURRENT VALUE
===            =========                                 =============

 1             INITIAL NO.  OF GRID POINTS (ACB)          11
 2             MAXIMUM NO.  OF GRID POINTS (ACB)          21
 3             MAXIMUM NO.  OF GRID POINTS (AMCB)         33
 4             TOLERANCE                                  0. 1D-04
```

ACB STRATEGY - INITIAL GUESS   ACB STRATEGY - INITIAL GUESS   ACB STRATEGY - INITIAL GUESS   ACB STRATEGY - INITIAL GUESS

0.00000	0.00000	0.00000	1.25115	0.08069	0.68982	1.89655	0.12898	0.12898	2.00008
0.10000	0.12511	0.00081	1.19501	0.30954	0.62084	1.91621	0.12898	0.12898	2.00008
0.20000	0.25023	0.00323	1.13888	0.52789	0.55185	1.93379	0.12898	0.12898	2.00008
0.30000	0.37534	0.00726	1.08275	0.73573	0.48287	1.94931	0.12898	0.12898	2.00008
0.40000	0.50046	0.01291	1.02662	0.93307	0.41389	1.96276	0.12898	0.12898	2.00008
0.50000	0.62557	0.02017	0.97048	1.11991	0.34491	1.97414	0.12898	0.12898	2.00008
0.60000	0.75069	0.02905	0.91435	1.29624	0.27593	1.98345	0.12898	0.12898	2.00008
0.70000	0.87580	0.03954	0.85822	1.46208	0.20695	1.99069	0.12898	0.12898	2.00008
0.80000	1.00092	0.05164	0.80208	1.61741	0.13796	1.99586	0.12898	0.12898	2.00008
0.90000	1.12603	0.06536	0.74595	1.76223	0.06898	1.99897	0.12898	0.12898	2.00008
1.00000	1.25115	0.08069	0.68982	1.89655	0.00000	2.00000	0.12898	0.12898	2.00008
	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00

ACB STRATEGY - PASVAR SOLN'S ACB STRATEGY - PASVAR SOLN'S ACB STRATEGY - PASVAR SOLN'S ACB STRATEGY - PASVAR SOLN'S

0.00000	0.00000	0.00000	1.81028	0.19796	0.17885	1.99412	0.21365	0.21365	2.23415
0.01667	0.03188	0.00006	1.75383	0.25796	0.17586	1.99432	0.21365	0.21365	2.23415
0.03333	0.06374	0.00023	1.70007	0.31611	0.17286	1.99451	0.21365	0.21365	2.23415
0.06667	0.12736	0.00091	1.60055	0.42720	0.16688	1.99488	0.21365	0.21365	2.23415
0.10000	0.19084	0.00204	1.51127	0.53193	0.16089	1.99524	0.21365	0.21365	2.23415
0.12500	0.23835	0.00319	1.45052	0.60672	0.15640	1.99550	0.21365	0.21365	2.23415
0.15000	0.28578	0.00459	1.39452	0.67855	0.15192	1.99575	0.21365	0.21365	2.23415
0.20000	0.38034	0.00815	1.29450	0.81426	0.14295	1.99624	0.21365	0.21365	2.23415
0.25000	0.47447	0.01271	1.20692	0.94052	0.13398	1.99670	0.21365	0.21365	2.23415
0.30000	0.56814	0.01828	1.12828	1.05842	0.12502	1.99712	0.21365	0.21365	2.23415
0.35000	0.66131	0.02485	1.05597	1.16871	0.11607	1.99752	0.21365	0.21365	2.23415
0.40000	0.75393	0.03241	0.98803	1.27192	0.10711	1.99789	0.21365	0.21365	2.23415
0.50000	0.93738	0.05048	0.85942	1.45845	0.08922	1.99853	0.21365	0.21365	2.23415
0.60000	1.11820	0.07244	0.73337	1.61938	0.07134	1.99906	0.21365	0.21365	2.23415
0.70000	1.29613	0.09824	0.60383	1.75457	0.05348	1.99947	0.21365	0.21365	2.23415
0.80000	1.47092	0.12780	0.46777	1.86294	0.03564	1.99977	0.21365	0.21365	2.23415
0.90000	1.64236	0.16107	0.32527	1.94315	0.01781	1.99994	0.21365	0.21365	2.23415
0.95000	1.72677	0.17907	0.25234	1.97234	0.00890	1.99999	0.21365	0.21365	2.23415
1.00000	1.81028	0.19796	0.17885	1.99412	0.00000	2.00000	0.21365	0.21365	2.23415
	0.27D-05	0.64D-06	0.71D-05	0.52D-05	0.33D-05	0.22D-06	0.35D-06	0.35D-06	0.12D-05

ACB STRATEGY - OPTIMALITY ACB STRATEGY - OPTIMALITY ACB STRATEGY - OPTIMALITY ACB STRATEGY - OPTIMALITY

0.00000	0.03307	1.98117	1.81028	1.88011	0.00000	1.65631
0.01667	0.06165	1.98110	1.79070	1.83970	-0.00182	1.65631
0.03333	0.09024	1.98098	1.77101	1.79573	-0.00364	1.65631
0.06667	0.14744	1 98063	1.73203	1.70004	-0.00727	1.65630
0.10000	0.20468	1.98010	1.69445	1.60121	-0.01091	1.65628
0.12500	0.24764	1.97958	1.66748	1.53063	-0.01364	1.65626
0.15000	0.29063	1.97897	1.64155	1.46646	-0.01637	1.65624
0.20000	0.37673	1.97745	1.59201	1.36269	-0.02184	1.65619
0.25000	0.46302	1.97552	1.54374	1.29137	-0.02730	1.65612
0.30000	0.54953	1.97316	1.49452	1.24726	-0.03277	1.65603
0.35000	0.63632	1.97037	1.44243	1.22490	-0.03824	1.65593
0.40000	0.72341	1.96712	1.38580	1.22032	-0.04371	1.65582
0.50000	0.89871	1.95914	1.25305	1.25455	-0.05466	1.65554
0.60000	1.07579	1.94902	1.08579	1.33467	-0.06562	1.65520
0.70000	1.25504	1.93645	0.87608	1.44247	-0.07658	1.65480
0.80000	1.43690	1.92107	0.62104	1.55021	-0.08755	1.65433
0.90000	1.62181	1.90247	0.32515	1.62810	-0.09852	1.65381
0.95000	1.71557	1.89179	0.16530	1 64913	-0.10400	1.65352
1.00000	1.81028	1.88011	0.00000	1.65631	-0.10949	1.65322
	0.30D-06	0.15D-05	0.79D-05	0.27D-04	0.15D-06	0.22D-05

AMCB STRATEGY - INITIAL GUESS			AMCB STRATEGY - INITIAL GUESS			AMCB STRATEGY - INITIAL GUESS			AMCB STRATEGY - INITIAL GUESS		
0.00000	0.00000	0.00000	1.81028	0.19796	0.17885	1.99412	1.81028	1.88011	0.21365	0.21365	2.23415
0.01667	0.03188	0.00006	1.75383	0.25796	0.17586	1.99432	1.79070	1.83970	0.21365	0.21365	2.23415
0.03333	0.06374	0.00023	1.70007	0.31611	0.17286	1.99451	1.77101	1.79573	0.21365	0.21365	2.23415
0.06667	0.12736	0.00091	1.60055	0.42720	0.16698	1.99488	1.73203	1.70004	0.21365	0.21365	2.23415
0.10000	0.19084	0.00204	1.51127	0.53193	0.16089	1.99524	1.69445	1.60121	0.21365	0.21365	2.23415
0.12500	0.23835	0.00319	1.45052	0.60672	0.15640	1.99550	1.66748	1.53063	0.21365	0.21365	2.23415
0.15000	0.28578	0.00459	1.39452	0.67855	0.15192	1.99575	1.64155	1.46646	0.21365	0.21365	2.23415
0.20000	0.38034	0.00815	1.29450	0.81426	0.14295	1.99624	1.59201	1.36269	0.21365	0.21365	2.23415
0.25000	0.47447	0.01271	1.20692	0.94052	0.13398	1.99670	1.54374	1.29137	0.21365	0.21365	2.23415
0.30000	0.56814	0.01828	1.12828	1.05842	0.12502	1.99712	1.49452	1.24726	0.21365	0.21365	2.23415
0.35000	0.66131	0.02485	1.05597	1.16871	0.11607	1.99752	1.44243	1.22490	0.21365	0.21365	2.23415
0.40000	0.75393	0.03241	0.98803	1.27192	0.10711	1.99789	1.38580	1.22032	0.21365	0.21365	2.23415
0.50000	0.93738	0.05048	0.85942	1.45845	0.08922	1.99853	1.25305	1.25455	0.21365	0.21365	2.23415
0.60000	1.11820	0.07244	0.73337	1.61938	0.07134	1.99906	1.08579	1.33467	0.21365	0.21365	2.23415
0.70000	1.29613	0.09824	0.60383	1.75457	0.05348	1.99947	0.87608	1.44247	0.21365	0.21365	2.23415
0.80000	1.47092	0.12780	0.46777	1.86294	0.03564	1.99977	0.62104	1.55021	0.21365	0.21365	2.23415
0.90000	1.64236	0.16107	0.32527	1.94315	0.01781	1.99994	0.32515	1.62810	0.21365	0.21365	2.23415
0.95000	1.72677	0.17907	0.25234	1.97234	0.00890	1.99999	0.16530	1.64913	0.21365	0.21365	2.23415
1.00000	1.81026	0.19796	0.17885	1.99412	0.00000	2.00000	0.00000	1.65631	0.21365	0.21365	2.23415
	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00

AMCB STRATEGY - PASVAR SOLN'S			AMCB STRATEGY - PASVAR SOLN'S			AMCB STRATEGY - PASVAR SOLN'S			AMCB STRATEGY - PASVAR SOLN'S		
0.00000	0.00000	0.00000	1.10832	1.19633	0.41232	1.96900	1.10832	1.26301	0.12701	1.14224	2.14911
0.01667	0.01896	0.00002	1.09638	1.21483	0.40537	1.97003	1.09624	1.26212	0.12701	1.14224	2.14911
0.03333	0.03791	0.00008	1.08455	1.23313	0.39843	1.97104	1.08401	1.26160	0.12701	1.14224	2.14911
0.06667	0.07577	0.00032	1.06120	1.26913	0.38455	1.97301	1.05906	1.26164	0.12701	1.14224	2.14911
0.10000	0.11359	0.00072	1.03819	1.30436	0.37069	1.97491	1.03341	1.26305	0.12701	1.14224	2.14911
0.12500	0.14193	0.00113	1.02113	1.33028	0.36029	1.97629	1.01369	1.26494	0.12701	1.14224	2.14911
0.15000	0.17024	0.00162	1.00421	1.35577	0.34991	1.97763	0.99354	1.26751	0.12701	1.14224	2.14911
0.20000	0.22676	0.00288	0.97072	1.40548	0.32915	1.98019	0.95190	1.27456	0.12701	1.14224	2.14911
0.25000	0.28317	0.00450	0.93756	1.45352	0.30841	1.98260	0.90836	1.28395	0.12701	1.14224	2.14911
0.30000	0.33944	0.00648	0.90459	1.49989	0.28770	1.98494	0.86283	1.29544	0.12701	1.14224	2.14911
0.35000	0.39556	0.00881	0.87168	1.54460	0.26700	1.98694	0.81521	1.30874	0.12701	1.14224	2.14911
0.40000	0.45154	0.01150	0.83870	1.58765	0.24633	1.98887	0.76542	1.32356	0.12701	1.14224	2.14911
0.50000	0.56300	0.01795	0.77213	1.66875	0.20506	1.99228	0.65911	1.35643	0.12701	1.14224	2.14911
0.60000	0.67375	0.02580	0.70416	1.74309	0.16387	1.99506	0.54363	1.39089	0.12701	1.14224	2.14911
0.70000	0.78372	0.03506	0.63429	1.81049	0.12277	1.99722	0.41914	1.42330	0.12701	1.14224	2.14911
0.80000	0.89284	0.04571	0.56231	1.87075	0.08176	1.99877	0.28625	1.44992	0.12701	1.14224	2.14911
0.90000	1.00106	0.05774	0.48824	1.92365	0.04084	1.99969	0.14604	1.46748	0.12701	1.14224	2.14911
0.95000	1.05481	0.06426	0.45049	1.94728	0.02041	1.99992	0.07364	1.47211	0.12701	1.14224	2.14911
1.00000	1.10832	0.07113	0.41232	1.96900	0.00000	2.00000	0.00000	1.47368	0.12701	1.14224	2.14911
	0.18D-06	0.24D-07	0.20D-06	0.28D-06	0.20D-06	0.29D-07	0.45D-06	0.17D-05	0.22D-07	0.42D-06	0.73D-07

AMCB STRATEGY - OPTIMALITY AMCB STRATEGY - OPTIMALITY AMCB STRATEGY - OPTIMALITY AMCB STRATEGY - OPTIMALITY

0.00000	0.04548	1.76196	1.10832	1.26301	0.00000	1.47368
0.01667	0.06294	1.76191	1.09624	1.26212	-0.00371	1.47367
0.03333	0.08041	1.76185	1.08401	1.26160	-0.00742	1.47366
0.06667	0.11534	1.76167	1.05906	1.26164	-0.01484	1.47361
0.10000	0.15030	1.76144	1.03341	1.26305	-0.02227	1.47353
0.12500	0.17652	1.76122	1.01369	1.26494	-0.02785	1.47345
0.15000	0.20275	1.76096	0.99354	1.26751	-0.03344	1.47336
0.20000	0.25525	1.76035	0.95190	1.27456	-0.04462	1.47310
0.25000	0.30780	1.75959	0.90836	1.28395	-0.05581	1.47278
0.30000	0.36041	1.75868	0.86283	1.29544	-0.06701	1.47238
0.35000	0.41308	1.75763	0.81521	1.30874	-0.07823	1.47191
0.40000	0.46584	1.75643	0.76542	1.32356	-0.08945	1.47137
0.50000	0.57161	1.75357	0.65911	1.35643	-0.11192	1.47007
0.60000	0.67780	1.75006	0.54363	1.39089	-0.13441	1.46847
0.70000	0.78449	1.74589	0.41914	1.42330	-0.15691	1.46658
0.80000	0.89175	1.74100	0.28625	1.44992	-0.17940	1.46440
0.90000	0.99966	1.73535	0.14604	1.46748	-0.20187	1.46193
0.95000	1.05389	1.73222	0.07364	1.47211	-0.21309	1.46059
1.00000	1.10832	1.72888	0.00000	1.47368	-0.22431	1.45917
	0.32D-10	0.19D-09	0.57D-06	0.15D-05	0.23D-06	0.15D-05

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* PARAMETER LIST *
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CASE PARAMETERS

KEY	PARAMETER	CURRENT VALUE
1	TIME	0.2300D+01
2	DISTANCE	0.2000D+01
3	U(ACC.)	0.1000D+02
4	U(BR.)	-.2000D+01
5	B(ACC.)	-.1000D-01
6	C(ACC.)	-.1000D-01
7	B(BR.)	-.1000D-01
8	C(BR.)	-.1000D-01
9	A	0.3000D-00
10	B	0.1400D-00
11	C	0.1600D-00
12	VMAX	0.1600D+01
13	MASS	0.1000D+01

GROUND PARAMETERS

KEY	PARAMETER	CURRENT VALUE	
1	EPSILON	0.5000D-00	
2	NO. OF B.P.'S	3	
		POSITION	GROUND
3	B.P. # 1	0.5000D-00	0.1000D+01
4	B.P. # 2	0.2000D+01	-.5000D-00
5	B.P. # 3	0.2500D+01	0.1000D+01

PASVAR PARAMETERS

KEY	PARAMETER	CURRENT VALUE
1	INITIAL NO. OF GRID POINTS (ACB)	11
2	MAXIMUM NO. OF GRID POINTS (ACB)	21
3	MAXIMUM NO. OF GRID POINTS (AMCB)	33
4	TOLERANCE	0.1D-04

AVCB STRATEGY - INITIAL GUESS			AVCB STRATEGY - INITIAL GUESS			AVCB STRATEGY - INITIAL GUESS			AVCB STRATEGY - INITIAL GUESS
0.00000	0.00000	0.00000	1.25115	0.08069	0.68982	1.89655	0.12898	0.12898	2.00008
0.10000	0.12511	0.00081	1.19501	0.30954	0.62084	1.91621	0.12898	0.12898	2.00008
0.20000	0.25023	0.00323	1.13888	0.52789	0.55185	1.93379	0.12898	0.12898	2.00008
0.30000	0.37534	0.00726	1.08275	0.73573	0.48287	1.94931	0.12898	0.12898	2.00008
0.40000	0.50046	0.01291	1.02662	0.93307	0.41389	1.96276	0.12898	0.12898	2.00008
0.50000	0.62557	0.02017	0.97048	1.11991	0.34491	1.97414	0.12898	0.12898	2.00008
0.60000	0.75069	0.02905	0.91435	1.29624	0.27593	1.98345	0.12898	0.12898	2.00008
0.70000	0.87580	0.03954	0.85822	1.46208	0.20695	1.99069	0.12898	0.12898	2.00008
0.80000	1.00092	0.05164	0.80208	1.61741	0.13796	1.99586	0.12898	0.12898	2.00008
0.90000	1.12603	0.06536	0.74595	1.76223	0.06898	1.99897	0.12898	0.12898	2.00008
1.00000	1.25115	0.08069	0.68982	1.89655	0.00000	2.00000	0.12898	0.12898	2.00008
	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00	0.00D-00

AVCB STRATEGY - PASVAR SOLN'S			AVCB STRATEGY - PASVAR SOLN'S			AVCB STRATEGY - PASVAR SOLN'S			AVCB STRATEGY - PASVAR SOLN'S
0.00000	0.00000	0.00000	1.60000	0.42990	0.18891	1.99344	0.18697	0.36038	2.23047
0.02500	0.04185	0.00010	1.53711	0.50321	0.18417	1.99377	0.18697	0.36038	2.23047
0.05000	0.08364	0.00039	1.47888	0.57370	0.17942	1.99408	0.18697	0.36038	2.23047
0.07500	0.12539	0.00088	1.42494	0.64156	0.17468	1.99439	0.18697	0.36038	2.23047
0.10000	0.16707	0.00156	1.37485	0.70700	0.16994	1.99469	0.18697	0.36038	2.23047
0.15000	0.25025	0.00351	1.28438	0.83123	0.16046	1.99526	0.18697	0.36038	2.23047
0.20000	0.33315	0.00624	1.20414	0.94751	0.15098	1.99581	0.18697	0.36038	2.23047
0.25000	0.41574	0.00974	1.13142	1.05665	0.14151	1.99631	0.18697	0.36038	2.23047
0.30000	0.49799	0.01401	1.06415	1.15926	0.13205	1.99679	0.18697	0.36038	2.23047
0.35000	0.57989	0.01905	1.00077	1.25578	0.12258	1.99723	0.18697	0.36038	2.23047
0.40000	0.66139	0.02485	0.94001	1.34649	0.11313	1.99764	0.18697	0.36038	2.23047
0.50000	0.82311	0.03873	0.82247	1.51125	0.09423	1.99836	0.18697	0.36038	2.23047
0.60000	0.98296	0.05562	0.70522	1.65413	0.07534	1.99895	0.18697	0.36038	2.23047
0.70000	1.14073	0.07548	0.58408	1.77476	0.05648	1.99941	0.18697	0.36038	2.23047
0.80000	1.29627	0.09826	0.45711	1.87221	0.03764	1.99974	0.18697	0.36038	2.23047
0.90000	1.44941	0.12393	0.32469	1.94539	0.01881	1.99993	0.18697	0.36038	2.23047
0.95000	1.52503	0.13784	0.25706	1.97259	0.00940	1.99998	0.18697	0.36038	2.23047
1.00000	1.60000	0.15245	0.18891	1.99344	0.00000	2.00000	0.18697	0.36038	2.23047
	0.15D-09	0.44D-10	0.40D-05	0.28D-05	0.83D-06	0.57D-07	0.42D-10	0.12D-05	0.30D-06

Appendix F.

Sample of PASVAR diagnostics (error messages) file.

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  JACOBIAN IS SINGULAR IN NEWTON STEP
  JACOBIAN IS SINGULAR IN NEWTON STEP
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  JACOBIAN IS SINGULAR IN NEWTON STEP
ERROR RETURN FROM DD04AD
THE PROGRAM ATTEMPTED TO USE A GRID WITH MORE THAN NMAX POINTS.

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